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Phase transition between three- and two-flavor QCD?

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Abstract. We explore the possibility that QCD may undergo a phase transition as a function of the strange quark mass. This would hint towards models with "spontaneous color symmetry breaking" in the vacuum. For two light quark flavors we classify possible colored quark–antiquark, diquark and gluon condensates that are compatible with a spectrum of integer charged states and conserved isospin and baryon number. The "quark mass phase transition" would be linked to an unusual realization of baryon number in QCD_2 and could be tested in lattice simulations. We emphasize, however, that at the present stage the Higgs picture of the vacuum cannot predict a quark mass phase transition – a smooth crossover remains as a realistic alternative. Implications of the Higgs picture for the high-density phase transition in QCD_2 suggest that this transition is characterized by the spontaneous breaking of isospin for nuclear and quark matter.

1 Introduction

Spontaneous color symmetry breaking by a colored quark–antiquark condensate in the vacuum has been proposed [1] as a "complementary" or "dual" picture for low energy QCD with three flavors of light quarks. This idea relies on the correspondence between a confinement and a Higgs description [2] and finds an analogy in high-density quark matter [3]. First dynamical investigations suggest that the color octet $\bar{q}q$-condensate is induced dominantly by instanton effects [4], with fermion fluctuation effects going into the same direction [5]. The phenomenological success of this description for realistic QCD with three flavors of light quarks [1] includes the hadronic and leptonic decays of the ρ-, K- and π-mesons, including explanations of vector dominance and the $\Delta I = 1/2$ rule for weak kaon decays. The coincidence of deconfinement with the high temperature chiral phase transition in QCD results from the melting of the octet condensate at high temperature [6].

The idea is intriguing enough that one may look for possible tests to verify or falsify this picture. Unfortunately, finding a clear-cut test is not so easy. The first is, of course, comparison with observation. On a rough level, the model actually does surprisingly well. Finding decisive quantitative precision tests is hindered at present by the lack of knowledge of the effective action relevant for low momentum scales. As long as the parameters in the effective action are not computed from QCD there remains substantial freedom to adapt parameters to observation, limiting the predictivity. Second, direct tests by lattice simulations require some thought since on a fundamental level gauge symmetries are never spontaneously broken and the Higgs picture is only an approximate language – which may nevertheless be very useful, as is well known from the electroweak gauge theory. In particular, the color octet $\bar{q}q$-bilinear has a zero expectation value in any gauge-invariant formulation. In lattice simulations a non-zero octet expectation value could be seen only in an appropriately gauge-fixed version. The difficulty of finding a direct simple lattice test is actually quite profound and can be traced back to the equivalence of the Higgs and confinement pictures. Needless to say that the identification of a suitable test quantity and its measurement by a simulation would be of great value.

As a third possibility, one may want to look more explicitly into the proposal that instanton dynamics is responsible for "spontaneous color symmetry breaking". Simulations based on instanton ensembles do not exhibit the full local color symmetry and therefore could, in principle, show an octet condensation. The problem with a first simulation [7] concerns the limited value of an instanton ensemble where the large-scale instantons are removed "by hand". These large-scale instantons are responsible for the octet condensation in the computation of [4]. In fact, the octet condensate is supposed to provide the infrared cut-off for the instanton ensemble since it leads to a nonvanishing gluon mass. In more explicit terms an instanton computation should proceed in two steps. For the functional integral over gauge fields and quarks one first performs the integration over gauge fields for fixed values of the fermion fields and approximates it by a suitable integral over instantons.[1] The second step solves the remain-

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[1] The integration over the fermionic zero modes may be included in the first step

ing functional integral over instanton collective coordinates and fermions by simulation. The first step needs the computation of instanton solutions in the presence of nonzero fermion-bilinear sources (currents) for the gauge field. For colored bilinears this introduces an important dependence of the effective instanton ensemble on the fermion bilinears which has not been taken into account so far. Including the corresponding interactions for large $\bar{q}q$ is crucial for the proposed IR cut-off [4] and mandatory for a test of this idea.

Waiting for a possible test along these lines one may investigate, as a fourth alternative, if a characteristic dependence on some "external parameters" like temperature or quark masses could lead to a test. These parameters are not easily varied in nature, but for lattice simulations this is less of a problem. For the temperature dependence the octet condensate actually does fine. It can explain [6] in a natural way the coincidence of chiral symmetry restoration and deconfinement at the same critical temperature and leads to a realistic $T_c \approx 170\,\mathrm{MeV}$ for three light quarks with equal mass. In the present paper we look for possible tests via the dependence of strong interaction physics on the quark masses.

The realization of a Higgs picture of the QCD vacuum depends strongly on the number of light quark flavors. Its original proposal relies on the equality of the number of light flavors and the number of colors which permits a "color-flavor-locked" [4] diagonal global $SU(3)$ symmetry. On the other side, no $\bar{q}q$-condensate is available for an infrared cut-off in pure QCD without quarks (gluodynamics). If the octet condensate picture [1] applies for three light quark flavors, one would expect an important qualitative change as the three equal quark masses increase. Beyond a critical quark mass the infrared cut-off of gluodynamics is expected to dominate, and a possible octet condensate should disappear. It is conceivable that this qualitative change becomes visible in lattice simulations as a phase transition or a relatively sharp crossover as a function of the quark mass. This could be tested once realistic quark masses can be attained in simulations with dynamical fermions.

For two light flavors or the limit of a large strange quark mass the situation is even more complicated. A simultaneous condensation of $\bar{q}q$-pairs and diquarks has been proposed for the vacuum of QCD_2 [8]. The diquark condensate introduces very interesting features which distinguish QCD_2 from the three-flavor case QCD_3. Notably, baryon number is spontaneously broken by the diquark condensate and replaced by a new type of conserved baryon charge B'. Whereas two of the six quarks carry $B' = 1$ and can be identified with the proton and neutron, the other four have $B' = 0$. This leads to an interesting Higgs description where quarks carry fractional quark-baryon charge $B_q = 1/3$ whereas they are integer charged with respect to B'. This particular feature of QCD_2 influences strongly the transition to the high-density and high-temperature state [8,9].

Lowering the strange quark mass from high values to zero moves us from QCD_2 to QCD_3. The different realization of baryon number in QCD_2 and QCD_3 suggests the possibility that a phase transition could occur at some critical strange quark mass $m_{s,c}$. For this critical value the baryon charge of the hyperons Σ and Λ switches from $B' = 0$ (for QCD_2) to $B = 1$ (for QCD_3). A similar change occurs for the effective strangeness quantum number. From the observed realization of B and S one concludes that $m_{s,c}$ should be higher than the physical value of m_s. A phase transition as a function of m_s can be studied by lattice simulations. Since the existence and the details of such a phase transition depend crucially on the condensates in the vacuum of QCD_2, in particular on the occurrence of diquark condensation, the Higgs picture of the QCD vacuum cannot clearly predict the presence of a transition at the present stage. Nevertheless, the observation of such a transition in lattice simulations would give a very important hint about the vacuum condensates in QCD!

In fact, in the absence of colored condensates one expects that the usual color singlet $\bar{q}q$-condensates change smoothly as a function of the quark mass. At least there is no obvious reason for a phase transition. The observation of a phase transition between QCD_3 and QCD_2 as a function of m_s or between QCD with three or two light flavors and gluodynamics could therefore be interpreted as a rather clear signal for the occurrence of other condensates. In particular, we will see that it could find a natural explanation within a Higgs picture with an octet condensation in the vacuum!

In this note we explore the possibility of testing the Higgs picture of the QCD vacuum by a phase transition as a function of m_s in more detail. For this purpose we present a systematic study of possible symmetry-breaking patterns in QCD_2 with two flavors. We restrict our discussion to vacuum states which preserve a global isospin symmetry such that the eight gluons transform as a triplet (ρ) two doublets ($K^*, \bar{K}^*$) and a singlet (ω). Similarly, two of the quarks should carry the quantum numbers of the proton and the neutron. Our classification of $\bar{q}q$, diquark and gluonic condensates carries over to the high-density state of QCD_2. Dynamical arguments point to a simultaneous condensation of $\bar{q}q$ and diquarks in the high-density state of QCD_2 [9]. This implements color-flavor locking for two flavors. Our discussion of arbitrary expectation values of isospin-conserving $\bar{q}q$- and diquark operators partly overlaps with other discussions of high-density condensates [10,3]. For the vacuum state we extend the discussion of [8] by the inclusion of other possible condensates.[2] Since all arguments presented here are solely based on symmetry properties they apply independently of a given dynamical scenario which would explain how the color symmetry-breaking condensates are generated.

Besides the possible phase transition between QCD_3 and QCD_2 as a function of the strange quark mass we are interested here in a second issue. We want to classify the characteristics of a possible high-density phase transition in QCD_2 in dependence on the baryon den-

[2] See also [11] for early discussions of the Higgs mechanism for $SU(3)$-gauge theories with fundamental scalars

sity, assuming[3] that a colored condensate occurs in the vacuum. For both purposes we first study possible condensates which might be relevant for the ground state or the high-density state of QCD_2. We begin in Sect. 2 with quark–antiquark condensates transforming as color octets. This discussion is extended in Sect. 3 to include diquark condensates whereas gluonic condensates in nontrivial color representations are added in Sect. 4.

In Sect. 5 we turn to the transition between QCD_2 and QCD_3 as the strange quark mass is lowered from very large values towards zero. If QCD_2 is characterized by a diquark condensate, a phase transition is plausible (but not necessary). The high-density phase transition in QCD_2 is discussed in Sect. 6. Its characteristics depend on the condensates in the vacuum, in particular on the question of a vacuum-diquark condensate. A continuous crossover to the high-density phase becomes possible if isospin is conserved in the high-density phase. We suggest that spontaneous breaking of isospin in the high-density phase leads to a true phase transition. The spectrum of excitations in nuclear and quark matter contains in this case a massless Goldstone boson a^0 responsible for superfluidity as well as two light pseudo-Goldstone bosons $a^{\pm}$ in addition to the pions. Furthermore, we present in Sect. 7 an at first only partially successful attempt to understand the ground state of pure QCD (no light quarks) in terms of color symmetry-breaking gluonic condensates. Finally, in Sect. 8 we discuss the transition to gluodynamics when all three light quarks get heavy simultaneously. In particular, we investigate the consequences of the dual description of the QCD-vacuum by a Higgs and confinement picture for the shape of the heavy quark potential. Our conclusions are presented in Sect. 9.

Our main findings are as follows.

(1) For QCD_2 the condensation of both a color octet quark–antiquark pair and an antitriplet (or sextet) diquark pair in the vacuum would lead to complete "spontaneous symmetry breaking" for the local color group [8]. All gluons acquire a mass. A residual global $SU(2)$ isospin symmetry and global baryon number B' for the proton and neutron remain preserved. The local abelian electromagnetic symmetry leads to integer charges Q for the quarks. In the Higgs picture the quarks are identified with baryons. Chiral symmetry is spontaneously broken and the baryons are massive. The massive gluons are integer charged and can be associated with eight vector mesons. Without the diquark condensate one of the gluons corresponding to the ω-meson would remain massless.

(2) As the strange quark is added and its mass lowered, two-flavor QCD transmutes into realistic QCD and finally into three-flavor QCD with three massless or very light quarks. For a very heavy strange quark the dynamics remains essentially the same as for two-flavor QCD. A new global symmetry is added, however, which acts on the new massive excitations. In the vacuum it corresponds to a new conserved strangeness quantum number S'. All s-quarks have $S' = -1$. The QCD vacuum with diquark condensation exhibits now three independent global symmetries (in the absence of electromagnetism) which correspond to Q, B', and S'.

(3) The standard symmetries B and S are not realized by the vacuum or high-density state of QCD_2 if all gluons acquire a mass from isospin-conserving scalar condensates. These generators are broken by the diquark condensate. The spontaneous breaking of B (or S) induces, however, no Goldstone boson since it is accompanied by the breaking of a local symmetry which is part of the color symmetry. The "would-be Goldstone boson" is eaten by the Higgs mechanism. (It is the longitudinal component of the λ_8-gluon or ω-meson.)

(4) For a light strange quark mass equal to the up and down quark mass one expects the unbroken global $SU(3)$-symmetry of QCD_3. In this case the color octet condensate gives mass to all gluons. In particular, the ω acquires a mass from the condensation of the strangeness components of the color octet quark–antiquark pair. No diquark condensate is possible in the vacuum since this would break baryon number. The preserved exact global symmetries correspond now to Q, B and S. The quantum numbers B and S for the fermions in QCD_3 differ from B' and S' for QCD_2 in presence of a diquark condensate. In this case one therefore expects a transition as a function of the strange quark mass m_s, from a diquark condensate for large m_s to an (almost) $SU(3)$-symmetric octet condensate for small m_s.

(5) The transition between QCD_3 and QCD_2 may be associated with a phase transition (as a function of m_s). We note that an intermediate state with both an antitriplet diquark condensate and non-vanishing strangeness components of the octet condensate would lead to a state with a different realization of the symmetries. One of the global symmetries (S) is broken in such a state, implying the existence of an exactly massless Goldstone boson and therefore superfluidity. The transition between large and small m_s could therefore either proceed directly by a first-order phase transition or pass by two transitions with an intermediate superfluid phase. As an alternative one may conceive an intermediate Coulomb-like phase where only the on-strange components of the octet condense and one gluon remains massless in the Higgs picture. As a general remark we recall, however, that the Higgs picture is only an approximation and may not be reliable for all questions. Since no global symmetries of the vacuum are altered as m_s switches from large to small values an analytic crossover remains possible as well.

(6) We suggest that the high-density transition to nuclear matter is characterized by a spontaneous breaking of the global isospin symmetry due to a dineutron condensate. In this event an exactly massless Goldstone boson a^0 exists in nuclear matter, corresponding to the spontaneous breaking of the global I_3-symmetry. (The latter is an exact symmetry even in presence of non-vanishing quark masses and electromagnetism.) In case of spontaneous isospin breaking, nuclear matter is a superfluid. In addition, two new

[3] Without a colored vacuum condensate this phase transition is extensively discussed in [10]

Table 1. States with conserved $SU(2)_I$

	Condensate		Superfluidity	Electric charge	Massless gluons
A	8_3	$\bar{\xi}_1$	no	integer	1
B	$8_3 + \bar{3}_1$	$\bar{\xi}_1, \bar{\delta}_1$	no	integer	0
C	$\bar{3}_1$	$\bar{\delta}_1$	no	integer	3
D	$8_3, 8_2$	$\bar{\xi}_1, \bar{\xi}_2$	no	integer	0
E	$8_3, 8_2, \bar{3}_1,$	$\bar{\xi}_1, \bar{\xi}_2, \bar{\delta}_1$	yes	integer	0
F	$8_3, 10$	$\bar{\xi}_1, \bar{a}$	no	fractional	0
G	$8_3, \bar{3}_1, 10$	$\bar{\xi}_1, \bar{\delta}_1, \bar{a}$	no	fractional	0

light scalars $a^{\pm}$ correspond to the breaking of the generators $I^{\pm}$. The mass of these pseudo-Goldstone bosons vanishes in the limit of equal up and down quark masses and in the absence of electromagnetism. This behavior is similar to the well-known pions. Spontaneous isospin breaking may occur also for the high-density "quark matter" phase of QCD_2.

Before starting with a more detailed description of the different possible "states" of QCD_2 in the main part of this paper, we present a brief summary in Table 1. Here the different possible condensates are denoted by their $SU(3)_c$-representation with the $SU(2)$-flavor representation as a subscript. The naming and details are explained in the main text. We do not list the usual color singlet $\bar{q}q$-condensate which may always accompany the other condensates. The standard picture of the QCD vacuum would have vanishing expectation values for all condensates listed in Table 1.

2 Quark–antiquark color octet

Let us start with a discussion of the color symmetry-breaking pattern induced by quark–antiquark condensates in the vacuum of QCD_2. This corresponds to the entry (A) in Table 1. With respect to the $SU(3)_c$ group a quark–antiquark pair can be in a singlet $\tilde{\varphi}$ or an octet $\tilde{\chi}$

$$\tilde{\varphi}^{(1)}_{ab} = \bar{\psi}_{\mathrm{L}\ ib}\ \psi_{\mathrm{R}\ ai} \quad , \quad \tilde{\varphi}^{(2)}_{ab} = -\bar{\psi}_{\mathrm{R}\ ib}\ \psi_{\mathrm{L}\ ai},$$
$$\tilde{\chi}^{(1)}_{ij,ab} = \bar{\psi}_{\mathrm{L}\ jb}\ \psi_{\mathrm{R}\ ai} - \frac{1}{3}\bar{\psi}_{\mathrm{L}\ kb}\ \psi_{\mathrm{R}\ ak}\ \delta_{ij},$$
$$\tilde{\chi}^{(2)}_{ij,ab} = -\bar{\psi}_{\mathrm{R}\ jb}\ \psi_{\mathrm{L}\ ai} + \frac{1}{3}\bar{\psi}_{\mathrm{R}\ kb}\ \psi_{\mathrm{L}\ ak}\ \delta_{ij}. \quad (1)$$

Here the color indices i, j run from 1 to 3 and the flavor indices $a, b = 1, 2$ denote the up and down quark. We consider vacuum expectation values of $\tilde{\varphi}$ and $\tilde{\chi}$ that lead to color-flavor locking where a diagonal $SU(2)_I$-subgroup of local $SU(2)_c$-color and global vector-like $SU(2)_F$-flavor remains unbroken. The unbroken global "physical" $SU(2)_I$ symmetry is associated with isospin. The decomposition of the $SU(2)_c \times SU(2)_F$-representations $\tilde{\varphi}, \tilde{\chi}$ with respect to $SU(2)_I$,

Table 2. Charges of the up and down quarks

	$\tilde{Q}$	Q_c	Q	I_3	S	B	B'	S'	Q'	
u_1	2/3	2/3	0	0	−1	1	0	0	1/6	Σ^0, Λ^0
u_2	2/3	−1/3	1	1	−1	1	0	0	7/6	Σ^+
u_3	2/3	−1/3	1	1/2	0	1	1	0	2/3	p
d_1	−1/3	2/3	−1	−1	−1	1	0	0	−5/6	Σ^-
d_2	−1/3	−1/3	0	0	−1	1	0	0	1/6	Σ^0, Λ^0
d_3	−1/3	−1/3	0	−1/2	0	1	1	0	−1/3	n

$$\tilde{\varphi} : (1, 1+3) \to 1+3,$$
$$\tilde{\chi} : (8, 1+3) \to 1+1+2+2+2+2+3+3+3 +4+4+5, \quad (2)$$

shows the existence of three singlets which can acquire a vacuum expectation value.

We will use a bosonized language where the quark–antiquark bilinears are replaced by scalar fields $\tilde{\varphi}^{(1)}_{ab} \to \sigma_{ab}$, $\tilde{\varphi}^{(2)}_{ab} \to \sigma^{\dagger}_{ab}$, $\tilde{\chi}^{(1)}_{ij,ab} \to \xi_{ij,ab}$, $\tilde{\chi}^{(2)}_{ij,ab} \to \xi^{*}_{ji,ba}$. The most general isospin-conserving expectation values are

$$\sigma_{ab} = \bar{\sigma}\delta_{ab},$$
$$\xi_{ij,ab} = \frac{1}{2\sqrt{6}}\bar{\xi}_1(\lambda^k)_{ij}(\tau^k)_{ba} + \frac{1}{6\sqrt{2}}\bar{\xi}_2(\lambda^8)_{ij}\delta_{ab}, \quad (3)$$

where $\bar{\sigma}, \bar{\xi}_1$ and $\bar{\xi}_2$ correspond to the three singlets in (2). Here τ^k are the Pauli matrices and λ^z the Gell-Mann matrices, with a sum $k = 1, ..., 3$. We note the presence of two color-breaking directions $\bar{\xi}_1, \bar{\xi}_2$.

The physical electric charge Q is composed of the quark charge $\tilde{Q}$ ($\tilde{Q} = -2/3$ for up, $\tilde{Q} = 1/3$ for down) and the color generator Q_c

$$Q = \tilde{Q} - Q_c = \tilde{Q} - \frac{1}{2}\lambda_3 - \frac{1}{2\sqrt{3}}\lambda_8. \quad (4)$$

It is easy to check that the quark and gluons have integer electric charge. We can also relate the physical electric charge to isospin (I_3) and interpret the color λ_8-generator in terms of standard baryon number B and strangeness S

$$Q = I_3 + \frac{1}{6}B - \frac{1}{2\sqrt{3}}\lambda_8 = I_3 + \frac{1}{2}(B+S) = I_3 + \frac{1}{2}B'. \quad (5)$$

Here we use a normalization where the baryon number[4] of the fermion fields is $B = +1$ and the "hyperons" Σ and Λ carry $S = -1$. With respect to $SU(2)_I, S, Q$ and B the six quarks carry the quantum numbers of the baryons (p, n, Λ, Σ) (see Table 2). We note that two-flavor QCD has two independent exact global symmetries in the presence of non-vanishing quark masses and the absence of electromagnetism. These are $B' = B + S$ and Q. In the presence of electromagnetic interactions the generator Q will be gauged.

[4] For a detailed discussion of the issue of baryon number see [1]. The quark baryon number B_q obeys $B_q = B/3$

Table 3. Charges of the gluons

	$\tilde{Q}$	Q_c	Q	I	I_3	S	B	B'	
A_3	0	0	0	1	0	0	0	0	ρ^0
$A_1+\mathrm{i}A_2$	0	−1	1	1	1	0	0	0	ρ^+
$A_1-\mathrm{i}A_2$	0	1	−1	1	−1	0	0	0	ρ^-
$A_4+\mathrm{i}A_5$	0	−1	1	1/2	1/2	1	0	1	K^{*+}
$A_4-\mathrm{i}A_5$	0	1	−1	1/2	−1/2	−1	0	−1	K^{*-}
$A_6+\mathrm{i}A_7$	0	0	0	1/2	−1/2	1	0	1	K^{*0}
$A_6-\mathrm{i}A_7$	0	0	0	1/2	1/2	−1	0	−1	$\overline{K}^{*0}$
A_8	0	0	0	0	0	0	0	0	ω

The surprising fact that baryons with strangeness $S = -1$ (i.e. Λ, Σ) are described by up and down quarks arises from the fact that the strangeness of the baryons has a color component according to (5). Actually, for the symmetry-breaking pattern that we describe here the isospin symmetry corresponds to a subgroup of the physical $SU(3)$ symmetry discussed for the three-flavor case in [1]. As compared to the three flavor case it is sufficient to leave out the states corresponding to the strange quark. Those correspond to the baryons Ξ and a baryon singlet S (see Table 4 below).

The gluons carry the quantum numbers of the ρ-, K^*- and ω-mesons. Again, they describe also states with strangeness. Their quantum numbers are displayed in Table 3.

As a consequence of the Higgs mechanism seven out of the eight gluons acquire a mass. This can be inferred by inserting the vacuum expectation values (3) in the covariant derivative for ξ,

$$\begin{aligned}\mathcal{L}_{\mathrm{kin},\xi} &= \hat{Z}(D^\mu\xi)^*_{ij,ab}(D_\mu\xi)_{ij,ab},\\ (D_\mu\xi)_{ij,ab} &= \partial_\mu\xi_{ij,ab} - \mathrm{i}gA_{ik,\mu}\xi_{kj,ab} + \mathrm{i}g\xi_{ik,ab}A_{kj,\mu},\\ A_{ij,\mu} &= \frac{1}{2}A^z_\mu(\lambda_z)_{ij}.\end{aligned} \tag{6}$$

With $(\lambda^k)_{ij}(\tau^k)_{ba} = 2\delta_{ia}\delta_{jb} - \delta_{ij}\delta_{ab}$ we find

$$\begin{aligned}M^2_\rho &= \frac{2}{3}\hat{Z}g^2|\bar{\xi}_1|^2,\\ M^2_{K^*} &= \hat{Z}g^2\left(\frac{1}{4}|\bar{\xi}_1|^2 + \frac{1}{12}|\bar{\xi}_2|^2\right),\\ M^2_\omega &= 0,\end{aligned} \tag{7}$$

where M_ρ denotes the mass of the ρ-triplet, M_{K^*} the one of the two isospin doublets $K^*, \bar{K}^*$ and M_ω concerns the isospin singlet ω. In addition to global isospin symmetry the expectation values $\bar{\xi}_1, \bar{\xi}_2$ leave a local $U(1)_8$-subgroup of color unbroken which corresponds to the generator λ_8. This is the reason for the massless ω-meson. In the language of hadrons we may associate the corresponding charge with a linear combination of strangeness S and baryon number B,

$$\lambda_8 = -\sqrt{3}S - \frac{2}{\sqrt{3}}B. \tag{8}$$

Table 4. Charges of the strange quarks

	$\tilde{Q}$	Q_c	Q	I_3	S	B	B'	S'	
s_1	−1/3	2/3	−1	−1/2	−2	1	−1	−1	Ξ^-
s_2	−1/3	−1/3	0	1/2	−2	1	−1	−1	Ξ^0
s_3	−1/3	−1/3	0	0	−1	1	0	−1	S^0

The unbroken local abelian symmetry is a direct consequence of the group structure and conserved isospin. All possible vacuum expectation values of quark–antiquark operators preserve the local $U(1)_8$ symmetry, which is a remnant of the color symmetry. Independent of the detailed dynamics one therefore expects for a state were only octets condense a massless gauge boson similar to the photon, but with a strong gauge coupling. It is conceivable that this symmetry is realized in the Coulomb phase. If true, the existence of a massless spin 1 state could be checked by a numerical simulation of QCD_2. We note that $\bar{\xi}_1$ leaves the additional $U(1)_8$-symmetry unbroken and therefore does not contribute to M_ω, whereas an unbroken gauged $SU(2)_c\times U(1)_c$ symmetry for $\bar{\xi}_1 = 0, \bar{\xi}_2 \neq 0$ forbids a contribution of $\bar{\xi}_2$ to M_ρ and M_ω. All masses are proportional to the strong gauge coupling g, with proportionality factor associated to the wave function renormalization $\hat{Z}$.

The expectation values $\bar{\sigma}, \bar{\xi}_1$ and $\bar{\xi}_2$ spontaneously break the chiral $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ symmetry, resulting in three (almost) massless pions. (There are no pseudo-Goldstone bosons transforming as kaons in the two-flavor picture.) Chiral symmetry breaking also gives a mass to the fermions which we associate with the baryon masses. We write the relevant Yukawa-type interaction in the form

$$\begin{aligned}\mathcal{L}_y &= \bar{\psi}_{ia}(h_\sigma\sigma_{ab}\delta_{ij} + h_\xi\xi_{ij,ab})\frac{1+\gamma_5}{2}\psi_{bj}\\ &\quad -\bar{\psi}_{ia}(h_\sigma\sigma^\dagger_{ab}\delta_{ij} + h_\xi\xi^*_{ji,ba})\frac{1-\gamma_5}{2}\psi_{bj}.\end{aligned} \tag{9}$$

The corresponding contributions to the masses of the nucleons and hyperons $\left(\Sigma^0 = \frac{1}{\sqrt{2}}(u_1 - d_2),\ \Lambda^0 = \frac{1}{\sqrt{2}}(u_1 + d_2)\right)$ are

$$\begin{aligned}M_n &= h_\sigma\bar{\sigma} - \frac{1}{3\sqrt{6}}h_\xi\bar{\xi}_2,\\ M_\Sigma &= h_\sigma\bar{\sigma} - \frac{1}{2\sqrt{6}}h_\xi\bar{\xi}_1 + \frac{1}{6\sqrt{6}}h_\xi\bar{\xi}_2,\\ M_\Lambda &= h_\sigma\bar{\sigma} + \frac{3}{2\sqrt{6}}h_\xi\bar{\xi}_1 + \frac{1}{6\sqrt{6}}h_\xi\bar{\xi}_2.\end{aligned} \tag{10}$$

If electromagnetic interactions are added to QCD, a linear combination $I_{3F} + \frac{1}{6}B$ of the previously global symmetries becomes gauged. As a consequence, color symmetry breaking by the octet condensate ξ leaves now two gauge bosons massless, corresponding to the two directions $I_{3F} + \frac{1}{6}B - I_{3c}$ and $\lambda_{8,c}$. A linear combination of them is the photon.

The appearance of the massless gluon of the unbroken $U(1)_8$-symmetry is special for the quark–antiquark condensates in QCD_2. This phenomenon does not happen for three light flavors. For $N_f = 2$ is connected to conserved strangeness and the fact that strangeness has only a contribution from the color generator λ_8 and baryon number. In contrast, for $N_f = 3$ the strangeness quantum number also receives a contribution form a flavor generator and "strangeness locking" permits the breaking of the local $U(1)_8$-symmetry. As long as we only consider quark–antiquark condensates, there is no way in two-flavor QCD of conserving isospin in the vacuum and giving mass to the $U(1)_8$-gluon. For $N_f = 2$ we still have, however, the possibility of condensation of diquarks or pure gluonic operators consistent with the physical isospin symmetry (see below). Again, this is different from three flavors. For $N_f = 3$ color-flavor locking implies that the physical $SU(3)$-representation for the gluon degrees of freedom coincides with the $SU(3)$-representation. All gluonic $SU(3)$-singlets must also be color $SU(3)_c$-singlets. Singlet operators of the type $F_{ij}^{\mu\nu}\ F_{ji,\mu\nu}$ do not affect the symmetry-breaking pattern. We conclude that despite many similarities between QCD_2 and QCD_3 the details of spontaneous color symmetry breaking depend critically on the number of light quarks.

We finally demonstrate the connection between integer electric charge and a massless ω-meson for QCD_2 without diquark condensates by a simple group-theoretical argument. The color generator λ_8 can be represented as a combination of isospin, baryon number and electric charge

$$\lambda_8 = 2\sqrt{3}\left(I_3 + \frac{1}{6}B - Q\right). \tag{11}$$

Quark–antiquark as well as gluonic operators conserve baryon number. This implies that all gluonic operators which conserve isospin and break the $U(1)_8$-symmetry necessarily violate electric charge. Indeed, the 10-dimensional $SU(3)_c$-representation (contained in the antisymmetric product of two octets) has a $SU(2)$-singlet which is charged with respect to $U(1)_8$ (see Sect. 4). An expectation value of this singlet would give a mass to the $U(1)_8$-gluon. In presence of the electromagnetic gauge interaction, however, such a vacuum would result in a modified charge seen by the photon, given by $Q' = \tilde{Q} - \frac{1}{2}\lambda_3$. (This corresponds to (4) without the piece from λ_8.) In consequence, the physical fermions would carry electric charges $(1/6, 7/6, 2/3, -5/6, 1/6, -1/3)$ (cf. Table 2). Our scenario for QCD_2 leads to three interesting alternatives: if isospin and B are conserved in the vacuum and all condensates are scalars, either diquark condensation occurs or there is a massless gluon or the electric charges of the baryons are unusual. These alternatives can be tested by lattice simulations. The perhaps favored alternative is the spontaneous breaking of baryon number in the vacuum by a diquark condensate, which we will discuss next.

3 Diquark condensates

We next turn to possible diquark condensates. They are expected to play a role at high baryon density. In contrast to QCD_3 they can also be relevant for the vacuum in QCD_2 [8]. The simultaneous octet and diquark condensation corresponds to the entry (B) in Table 1, whereas a pure diquark condensation corresponds to (C). Scalar diquarks are in the antisymmetric product of two quark fields. With respect to $SU(3)_c \times SU(2)_F$ they transform as $(\bar{3}, 1) + (6, 3)$. We first consider the antitriplet

$$(\tilde{\delta}_{\mathrm{L,R}})_i = (\psi_{\mathrm{L,R}})_{aj\beta} c^{\beta\gamma} (\psi_{\mathrm{L,R}})_{bk\gamma} \epsilon_{ijk}\epsilon_{ab}, \tag{12}$$

with β, γ spinor indices and $c^{\beta\gamma}$ the antisymmetric charge conjugation matrix. The corresponding scalar fields $\delta_{\mathrm{L,R}} \sim \tilde{\delta}_{\mathrm{L,R}}$ transform as a $\bar{3}$ under color and a singlet under flavor. Being flavor singlets, the expectation values of $\delta_{\mathrm{L,R}}$ cannot contribute to the breaking of the global $SU(2)_F$-flavor symmetry or the chiral symmetry $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$. Unlike the case for three flavors, they cannot induce color flavor locking for two-flavor QCD. Nevertheless, they can contribute to the breaking of color. In the presence of octet condensates $\bar{\xi}_1, \bar{\xi}_2$ the expectation value of $\delta_{\mathrm{L,R}}$ presumably favors the isospin-conserving direction.[5] There is one isospin singlet for both δ_{L} and δ_{R} and parity is conserved for

$$\delta_{\mathrm{L}i} = \delta_{\mathrm{R}i} = \bar{\delta}_1 \delta_{i3}. \tag{13}$$

In addition to the global $SU(2)_F$ symmetry this expectation value preserves a local $SU(2)_c$-subgroup of color. Therefore $\bar{\delta} \neq 0$ contributes to the mass of the K^*- and ω-vector mesons, but not to the ρ-mesons. From the covariant kinetic term

$$\begin{aligned}\mathcal{L}_{\mathrm{kin},\delta} &= Z_\delta\{(D^\mu\delta_{\mathrm{L}})_i^*(D_\mu\delta_{\mathrm{L}})_i + (D^\mu\delta_{\mathrm{R}})_i^*(D_\mu\delta_{\mathrm{R}})_i\},\\ (D_\mu\delta)_i &= \partial_\mu\delta_i + \mathrm{i}g\delta_j A_{ji,\mu},\end{aligned} \tag{14}$$

one finds

$$\begin{aligned}M_\rho^2 &= 0,\\ M_{K^*}^2 &= \frac{1}{2}Z_\delta g^2|\bar{\delta}_1|^2,\\ M_\omega^2 &= \frac{2}{3}Z_\delta g^2|\bar{\delta}_1|^2.\end{aligned} \tag{15}$$

The expectation value (13) induces a spontaneous breaking of baryon number. It also breaks the local $U(1)_8$-symmetry which is preserved by $\bar{\xi}_1$ and $\bar{\xi}_2$. In consequence the ω-meson related to the λ_8-generator of $SU(3)_c$ becomes massive in a situation where $\bar{\xi}_1 \neq 0, \bar{\xi}_2 \neq 0, \bar{\delta} \neq 0$. In this setting all gluons have acquired a mass. This is easily seen by adding the contributions (7) and (15). A diquark condensate in the vacuum of QCD_2 is possible

[5] We note the possibility of an isospin violating alignment of the expectation values of $\delta_{\mathrm{L,R}}$ which breaks color completely. The orthogonal expectation values $\delta_{\mathrm{L}i} = \bar{\delta}_{\mathrm{L}}\delta_{i1}, \delta_{\mathrm{R}i} = \bar{\delta}_{\mathrm{R}}\delta_{i2}$ induces a gluon mass matrix $M_{yz}^2 = Z_\delta g^2\bar{\delta}^2\left(\frac{4}{3}\delta_{yz} + \frac{2}{\sqrt{3}}d_{yz8}\right)$ or $M_\rho^2 = 2Z_\delta g^2\bar{\delta}^2,\ M_{K^*}^2 = Z_\delta g^2\bar{\delta}^2, M_\omega^2 = \frac{2}{3}Z_\delta g^2\bar{\delta}^2$. All gluons are massive, with an order $M_\rho^2 > M_{K^*}^2 > M_\omega^2$ reversed as compared to the realistic QCD vacuum. For this alignment the standard parity transformation $\delta_{\mathrm{L}} \leftrightarrow \delta_{\mathrm{R}}$ is spontaneously broken and one has to investigate if another parity like discrete symmetry survives

since it preserves a new color-flavor-locked baryon number

$$B' = B_q - \frac{1}{\sqrt{3}}\lambda_8, \tag{16}$$

with $B_q = B/3$ the quark baryon number. In terms of the hadronic quantum numbers B and S this reads

$$B' = B + S. \tag{17}$$

In consequence, switching on the expectation value $\bar{\delta}$ preserves the total number of conserved global abelian symmetries. It shifts the global charge from B to $B+S$ and breaks the local $U(1)_8$ symmetry. We conclude that the formation of a $\bar{\delta}$ condensate does not lead to a massless Goldstone boson and to superfluidity. Regions in the QCD phase diagram with $\bar{\delta} = 0$ and $\bar{\delta} \neq 0$ can be analytically connected. The formation of a $\bar{\delta}$-condensate looks similar to the phase transition in the abelian Higgs model or to the onset of superconductivity by the formation of Cooper pairs. Only the photon is replaced by the ω-meson. If, in addition, one gauges the electromagnetic $U(1)$ symmetry, the true photon remains massless since the δ-condensate carries zero electric charge. Only one of the two massless gauge bosons for $\bar{\delta} = 0$ gets a mass for $\bar{\delta} \neq 0$.

The diquark contribution to the baryon masses follows from the Yukawa coupling

$$\mathcal{L} = h_\delta \epsilon_{ijk}\epsilon_{ab}(\delta^*_{\mathrm{L}i}\psi_{\mathrm{L}aj}c\psi_{\mathrm{L}bk} + \mathrm{L} \leftrightarrow \mathrm{R} + \mathrm{c.c.}). \tag{18}$$

This yields a Majorana-type mass term for the hyperons Σ and Λ

$$\mathcal{L} = h_\delta \bar{\delta}_1 \{ \Lambda^0_\mathrm{L} c \Lambda^0_\mathrm{L} - (\Sigma^+_\mathrm{L} c \Sigma^-_\mathrm{L} + \Sigma^-_\mathrm{L} c \Sigma^+_\mathrm{L} + \Sigma^0_\mathrm{L} c \Sigma^0_\mathrm{L}) \\ + \mathrm{L} \leftrightarrow \mathrm{R} + \mathrm{c.c.} \}. \tag{19}$$

We next turn to the possible condensation of the color sextet diquark. The sextet diquark is symmetric in both color and flavor indices

$$(\tilde{\beta}_{\mathrm{L,R}})_{ij,ab} \\ = \frac{1}{\sqrt{2}} \left\{ (\psi_{\mathrm{L,R}})_{ai\beta} c^{\beta\gamma} (\psi_{\mathrm{L,R}})_{bj\gamma} + (\psi_{\mathrm{L,R}})_{aj\beta} c^{\beta\gamma} (\psi_{\mathrm{L,R}})_{bi\gamma} \right\}. \tag{20}$$

The associated scalar field $\beta^{(\mathrm{L,R})}_{ij,ab}$ carries again two units of baryon number $B = 2$ and transforms under $SU(2)_I$ as $1+2+3+3+4+5$. It therefore contains one singlet

$$\beta^{(\mathrm{L})}_{ij,ab} = \beta^{(\mathrm{R})}_{ij,ab} = \frac{1}{2\sqrt{2}} \bar{\beta}(\delta_{ia}\delta_{jb} + \delta_{ja}\delta_{ib}). \tag{21}$$

The contribution of β to the mass of the gauge bosons reads (with wave function renormalization Z_β)

$$M^2_\rho = 2 Z_\beta g^2 \bar{\beta}^2, \quad M^2_{K^*} = Z_\beta g^2 \bar{\beta}^2, \\ M^2_\omega = \frac{2}{3} Z_\beta g^2 \bar{\beta}^2. \tag{22}$$

All local symmetries are spontaneously broken and all gauge bosons are massive. As for the case of the δ-condensate, the β-condensate leaves the global symmetry associated to $B+S$ unbroken. Again, a β-condensate does not lead to a massless Goldstone boson and to superfluidity. Actually, the fields $\bar{\delta}$ and $\bar{\beta}$ in (13) and (21) carry the same abelian quantum numbers and are both isospin singlets. As before, a gauged electromagnetic symmetry remains preserved by a β-condensate. In the presence of an octet condensate the antitriplet and sextet can mix. More precisely a color octet and antitriplet diquark condensate induce a sextet diquark condensate by a linear term in the effective potential for the sextet. This term is generated by a cubic interaction between χ, δ and β. Similarly, $\langle\chi\rangle \neq 0$ and $\langle\beta\rangle \neq 0$ induce $\langle\delta\rangle \neq 0$.

In summary, the Higgs picture for two-flavor QCD differs in two important aspects from the three-flavor case. First, the vacuum either admits diquark condensation or one finds a massless gauge boson in the vacuum (in addition to the photon). This holds if electric charges are integer and isospin is conserved. Second, there is no massless Goldstone boson in presence of diquark condensates. Therefore no superfluidity occurs in the high-density phase if isospin is conserved.

4 Gluonic condensates

We conclude the discussion of isospin-conserving states in QCD_2 by a brief listing of possible color-breaking gluon condensates. As long as they conserve $SU(2)_I$ and $B+S$, they are necessarily induced in presence of color-breaking quark–antiquark and diquark condensates. The simplest $SU(2)_I$-singlet operator is found in the octet

$$f_{ij} \sim \left(F^{\mu\nu}_{ik} F_{kj,\mu\nu} - \frac{1}{3} F^{\mu\nu}_{lk} F_{kl,\mu\nu} \delta_{ij} \right). \tag{23}$$

With respect to $SU(2)_I$ it decomposes as $8 \to 1+2+2+3$, with a possible singlet expectation value

$$\langle f_{ij} \rangle = \bar{f}(\lambda_8)_{ij}. \tag{24}$$

This remains neutral with respect to $U(1)_8$ and therefore cannot break this local symmetry. Nevertheless, it gives a contribution to M_{K^*}, i.e. it contributes

$$M^2_\rho = 0\ ,\ M^2_\omega = 0\ ,\ M^2_{K^*} = \frac{3}{2} Z_f g^2 \bar{f}^2. \tag{25}$$

The other non-trivial candidate in the (symmetric) product of two-field strength tensors is the 27-dimensional representation

$$s_{ijkl} \sim F^{\mu\nu}_{ij} F_{kl,\mu\nu} - \frac{1}{3} \left(F^{\mu\nu}_{im} F_{ml,\mu\nu} \delta_{jk} \right. \\ \left. + F^{\mu\nu}_{km} F_{mj,\mu\nu} \delta_{il} - \frac{1}{3} F^{\mu\nu}_{mn} F_{nm,\mu\nu} \delta_{ij} \delta_{kl} \right), \\ s_{ijjl} = s_{ijki} = 0. \tag{26}$$

It contains one $SU(2)$-singlet which is again invariant with respect to $U(1)_8$ and contributes to the gluon masses similar to (25).

The lowest dimension $SU(3)_c$-representation which contains a $SU(2)_c$-singlet with non-zero $U(1)_8$-charge and which has triality zero is the complex 10. It is contained in the antisymmetric product of two octets. A scalar formed from glue in the $10+\bar{10}$ representation is

$$a_{ijkl} \sim F^{\mu\nu}_{im} F_{mj,\nu}{}^{\rho} F_{kl,\rho\mu} - F^{\mu\nu}_{km} F_{ml,\nu}{}^{\rho} F_{ij,\rho\mu} - \frac{1}{3} F^{\mu\nu}_{nm} F_{mn,\nu}{}^{\rho} F_{kl,\rho\mu}\delta_{ij} + \frac{1}{3} F^{\mu\nu}_{nm} F_{mn,\nu}{}^{\rho} F_{ij,\rho\mu}\delta_{kl}, \quad (27)$$

with $a_{ijkl} = -a_{klij}$, $a_{ijjl} = a_{ljkl} = 0$. An expectation value of the $SU(2)$-singlets in a_{ijkl} would break $U(1)_8$ and give a mass to the corresponding gauge boson. A condensate of such a higher-order gluonic operator would also break $B+S$. Only $SU(2)_I$ and baryon number remain as unbroken global symmetries of the vacuum if such operators acquire an expectation value. After a coupling to electromagnetism the baryons would get the non-integer charges corresponding to $Q' = \tilde{Q} - \frac{1}{2}\lambda_3$ (see Table 2). In the presence of such a $B+S$-violating condensate the occurrence of a diquark condensate in the high-density phase would break the only global symmetry, i.e. baryon number. ($B+S$ is not "available" for a residual global symmetry any more.) The resulting Goldstone boson would lead to superfluidity. For completeness we have listed the characteristics of a-condensates in Table 1 (entries F, G). We will not consider them any further since fractional electric charges in the vacuum seem not very attractive.

5 Transition between QCD_3 and QCD_2

The vacuum of QCD with three light flavors of quarks exhibits the vector-like $SU(3)$-symmetry of the "eightfold way", if the quark masses are all equal. No diquarks can condense since this would break either baryon number or the $SU(3)$ symmetry. The only non-trivial condensate is a color octet quark–antiquark pair [1]. The ground state of realistic QCD presumably resembles three-flavor QCD and does not exhibit a diquark condensate either. This follows from the fact that the global symmetries corresponding to baryon number and strangeness are realized in a standard way. Both B and S are conserved quantum numbers, in contrast to the case of diquark condensation.

For a small mass difference between the strange quark and the two light quarks we expect a splitting of the masses of particles within a given $SU(3)$-multiplet according to their isospin representation. The color singlet and octet $\bar{q}q$-condensates involving strange quarks add to (3) a contribution[6]

$$\Delta\sigma_{ab} = \bar{\sigma}_s \delta_{a3}\delta_{b3},$$
$$\Delta\xi_{ij,ab} = \frac{1}{2\sqrt{6}}\bar{\xi}_3(\lambda^4_{ij}\lambda^4_{ba} + \lambda^5_{ij}\lambda^5_{ba} + \lambda^6_{ij}\lambda^6_{ba} + \lambda^7_{ij}\lambda^7_{ba}) - \frac{1}{3\sqrt{2}}\bar{\xi}_4\lambda^8_{ij}\delta_{a3}\delta_{b3}. \quad (28)$$

[6] We have omitted here one more electrically neutral $SU(2)_I$-singlet contained in the color octet, ξ_5. The expectation value $\bar{\xi}_5$ vanishes in the limit of equal quark masses

The additional condensates $\bar{\sigma}_s, \bar{\xi}_3, \bar{\xi}_4$ involve the third flavor index[7] and will be denoted as "strange components" of the color singlet and octet. Combining the "strange" and "non-strange" octet contributions to the vector meson masses one obtains

$$M^2_\rho = \hat{Z}g^2\left(\frac{2}{3}|\bar{\xi}_1|^2 + \frac{1}{3}|\bar{\xi}_3|^2\right),$$
$$M^2_{K^*} = \hat{Z}g^2\left(\frac{1}{4}|\bar{\xi}_1|^2 + \frac{1}{12}|\bar{\xi}_2|^2 + \frac{1}{2}|\bar{\xi}_3|^2 + \frac{1}{6}|\bar{\xi}_4|^2\right) + \frac{3}{2}Z_f g^2 \bar{f}^2,$$
$$M^2_\omega = \hat{Z}g^2|\xi_3|^2. \quad (29)$$

In the $SU(3)$-symmetric limit $\bar{\xi}_1 = \bar{\xi}_2 = \bar{\xi}_3 = \bar{\xi}_4, \bar{f} = 0$ the masses in the vector meson octet are all degenerate [1]. We observe that for $\bar{\xi}_1 = \bar{\xi}_3$ the masses of the ρ-mesons and ω-meson remain equal. For $m_s \neq m_{u,d}$ one expects that also colored gluon condensates of the type (24) are induced. If the octet condensate $\sim \bar{\xi}_3 - \bar{\xi}_1$ is suppressed as compared to the other $SU(3)$-breaking condensates, we find the phenomenologically interesting relation $M^2_\omega \approx M^2_\rho, M^2_{K^*} > M^2_\rho$.

It is, of course, possible to move gradually from three-flavor QCD to two-flavor QCD by increasing the mass of the strange quark. In this process the various $SU(2)_I$-singlets acquire different expectation values. For a small deviation from $SU(3)$ this leads to a mass splitting in the $SU(3)$-multiplets according to strangeness. In the limit $m_s \to \infty$ the condensates involving strange quarks vanish. In this limit the ω-meson would become massless in the (naive) Higgs picture for any vacuum without a diquark condensate. In this case it is conceivable, but not likely that a phase transition occurs as a function of m_s where the local $U(1)_8$-symmetry gets restored for large enough m_s and the ω-meson becomes massless. At this point the reader should be warned, however, that the naive Higgs picture with bosonic propagator approximated by $(q^2 + m^2)^{-1}$ could be quite misleading in presence of strong couplings. It is well possible that the propagator for small momenta and $m^2 = 0$ looks very different from $1/q^2-$, e.g., $1/q^4-$ such that the analytic continuation to Minkowski space has no pole. In this case no propagating gluon-type degree of freedom would occur and there would be no massless ω-meson. In this sense the "naive" Higgs picture underlying our present discussion may only be valid as long as the effective bosonic mass m^2 is large enough. In view of the fact that couplings are strong and no global symmetries are altered for $m_s \to \infty$ we find a phase transition not very likely in the absence of a diquark condensate in the QCD_2 vacuum.

From the point of view of a possible phase transition the perhaps more interesting alternative is characterized by a vacuum-diquark condensate (13) in the two-flavor limit $m_s \to \infty$. In this event two scenarios imply a phase transition between QCD_3 and QCD_2 as a function of m_s.

[7] Remember that in (3) the flavor index takes only the values 1,2 whereas now it runs from 1 to 3

Either the diquark condensation sets in in competition to the "strange" octet components. At the transition the conserved quantum numbers jump from B and S to the new charges B' and S'. Or their could be an intermediate phase with both diquarks and all components of the $q\bar{q}$-octet condensing. This intermediate phase would be signalled by superfluidity connected to the Goldstone boson arising from a reduction of the total number of global symmetries. In the latter case a phase transition would be mandatory since the global symmetries change as a function of m_s.

Keeping in mind our cautious remarks we next investigate the transition between large and small m_s within the naive Higgs picture. From the point of view of QCD_2 we may first add a heavy strange quark to QCD_2. This will not affect the low energy dynamics and the condensates of QCD_2. It introduces, however, a new global symmetry with conserved quantum number S' for the heavy quarks, i.e. $S' = -1$ for all strange quarks and $S' = 0$ for up and down quarks. The quantum numbers of the s-quark states in the hadronic language can be found in Table 4, which supplements Table 2.

We start with the case where the vacuum of QCD_2 exhibits a non-vanishing diquark condensate. In the presence of this diquark condensate the conserved quantum numbers for large m_s are Q, B' and S'. For small m_s, the vacuum must switch, however, to a state which conserves Q, B and S, similar to QCD_3. As we have mentioned, such a state is not compatible with a diquark condensate. The phase transition as a function of m_s corresponds therefore to the disappearance of the diquark condensate and to an associated change in quantum numbers of states in the low energy spectrum. It is supposed to happen for some critical value $m_{s,c}$. This change in the quantum numbers concerns all "strange baryons", i.e. not only the ones corresponding to strange quarks (Table 4), but also four of the "light quark states" listed in Table 2. It is noteworthy that the proton and neutron or the ρ-mesons are not affected by this switch in quantum numbers.

The switch in quantum numbers from B' and S' to B and S may either proceed directly or via an intermediate superfluid or Coulomb-like phase. The reason is the mismatch in quantum numbers for the diquark condensate and the "strange" component of the octet $\bar{q}q$-condensate. A transition where a non-zero diquark condensate is replaced by non-zero strangeness components of the octet condensate at the critical strange quark mass leads to a jump in the conserved charges. If the ω-meson is massless for $m_{s,c}$ or even for a whole interval of the strange quark mass, no discontinuity is necessary. In the intermediate Coulomb phase the $U(1)_8$ local gauge supersymmetry associated to the ω-meson allows one to rotate freely between (B, S) and (B', S'). Both pairs of quantum numbers are conserved. The limits of large and small m_s outside this "Coulomb" region correspond to different lockings of the λ_8 generator, whereas in the Coulomb region it is unlocked. (Recall that beyond the naive Higgs picture the "Coulomb region" may not have a massless gluonic excitation.) If the change from the diquark condensate to the condensation of the strange components of the octet proceeds without an intermediate "Coulomb region", it may be most likely a discontinuous first-order phase transition. Finally, if there is an intermediate range of m_s where both the diquark condensate and the strange component of the octet condensate are non-zero, their simultaneous presence leads to the breaking of one of the two global symmetries. This corresponds to an intermediate superfluid phase.

Finally, we discuss the perhaps less likely possibility of a phase transition without diquark condensation in the vacuum of QCD_2. In this case one starts at high m_s with a Coulomb-like phase where both (B', S') and (B, S) are simultaneously conserved. The transition at $m_{s,c}$ is then characterized by the onset of a non-vanishing vacuum expectation value of the strange components of the octet. This gives a mass to the ω-meson and the transition shows similarities to the transition in a superconductor.

All versions of the transition show a remarkable behavior of the fate of the strange baryons as m_s increases from zero to large values. Only the Ξ-baryons and the singlet S^0 become very heavy and decouple from the effective low energy theory, similar to the baryons involving the heavy quarks c, b and t [1]. This decoupling does not take place for the baryons Σ and Λ^0 and for the K^*-vector mesons. These particles remain in the spectrum of light particles even in the limit $m_s \to \infty$. If diquark condensation sets in for large m_s, their properties become unusual since they cannot be built any more from a finite number of quarks in the non-relativistic quark model [8].

This unusual behavior of the K^*, Σ and Λ particles may give a hint for where to look for a possible transition in a lattice simulation. Indeed, the masses and couplings of the pseudoscalars (π, K, η, η'), the ρ-vector mesons and the nucleons (p, n) may depend rather smoothly on m_s (with the possibility of minor jumps in case of a discontinuous transition). In particular, we have seen that the masses of p, n, ρ, π are not affected by a diquark condensate. The qualitative changes between $m_s < m_{s,c}$ and $m_s > m_{s,c}$ are rather expected in the (K^*, Σ, Λ) sector. Here one may find some unusual behavior as the mass difference $m_s - m_q$ increases from (realistic) values within the approximate validity of the SU(3) flavor symmetry to large values. (Note that the relevant parameter is actually $m_s - m_q$ with m_q the mass of the up/down quark. It is important to keep this in mind since lattice simulations with very small m_q are not feasible at the present moment.) The present simulations with three dynamical quarks [12] are presumably within the range of approximate $SU(3)$ symmetry. It would be interesting to increase m_s with fixed m_q and to monitor the behavior of (K^*, Σ, Λ).

We observe, however, that for $m_s < m_{s,c}$ the "dressed gluons" K^* can mix with the corresponding quark–antiquark vector states $\sim \bar{q}\gamma^\mu q$ and similar for the "dressed quarks" Σ, Λ which can mix with three quark states.[8] This mixing may be forbidden for $m_s > m_{s,c}$, as in the case of diquark condensation. The correlation functions for the

[8] See [1] for a detailed discussion of the quantum numbers of the dressed gauge-invariant states

Table 5. Possible isospin-conserving high-density transition in QCD_2

Vacuum	High density	"Phase" transition	High density superfluidity	Massless gluon in vacuum
8	$8+\bar{3},6$	abelian Higgs	no	yes
$8+\bar{3},6$	$8+\bar{3},6$	continuous or first order	no	no

usual $\bar{q}\gamma^\mu q$ or qqq operators may therefore not reflect any more the states that we have denoted by (K^*, Σ, Λ) in the Higgs picture. This feature may render a direct observation of a transition rather difficult as long as one concentrates on correlation functions of operators like $\bar{q}\gamma^\mu q$ and qqq. In case of diquark condensation in the vacuum a more direct access to the particular features of such a ground state would become possible if local gauge-invariant operators corresponding to the states K^*, Σ, Λ can be constructed. These must be non-linear in the quark and gluon fields since no mesonic state with half integer isopin can be constructed as a finite power of quark and gluon fields. This also holds for Σ and Λ which have even isospin and should only involve contributions with an odd number of fermions.

6 High-density phase transition in QCD_2

The possible phase transitions between the vacuum and the high-density state of QCD_2 depend on the condensates in the vacuum and the high-density phase. We summarize in Table 5 the possibilities for the case of an isospin-conserving high-density phase with octet $\bar{q}q$- and diquark condensate. The global symmetries of the high-density state of QCD_2 and the vacuum are the same. In this event a continuous transition becomes possible. We distinguish the two alternatives without and with a diquark condensate in the vacuum ((A) and (B) in Table 1). For the latter case there is no obvious reason for a phase transition. On the other hand, if the transition to the high-density phase is characterized by the onset of diquark condensation one may expect a transition similar to the abelian Higgs model. This could be a first-order transition, but a second-order transition or a continuous crossover are also conceivable [13]. For a weak enough transition one expects the same universality class as for type II superconductors.

One may argue, however, that these scenarios are too simple. It seems likely that effective attractive interactions between the nucleons lead to an instability of the isospin-conserving states discussed so far. This instability is related to the pairing of nucleons. There is no contribution to a Majorana-like mass term for the proton and neutron from the diquark condensates $\bar{\delta}$ (cf. (19) or $\bar{\beta}$). In fact, s-wave and spin 0 nucleon pairs belong to an isospin triplet. This follows from the Pauli principle since the color part of the corresponding diquark operator is symmetric (proton and neutron are both quarks of the third color, see Table 2) and the spin part is antisymmetric. The dinucleon operator must therefore be symmetric in flavor, and this corresponds to an isospin triplet. Since the isospin singlet diquarks $\bar{\delta}$ and $\bar{\beta}$ cannot stabilize an instability in the nucleon pair channel, it seems likely that the condensation of nucleon pairs or diquarks of the third color produces the stabilizing gap.

We suggest that isospin is spontaneously broken in the high-density phase of QCD_2 as well as in nuclear matter. A candidate is a condensation of a diquark

$$\langle d_3 c d_3 \rangle \neq 0. \tag{30}$$

In the language of baryons this corresponds to a dineutron condensate. Such a condensate has interesting consequences. First, the spontaneous breaking of isospin produces three massless Goldstone bosons. In contrast to the pions they are scalars with the quantum numbers of the a-mesons. In the presence of a non-vanishing mass difference for the (current) mass of the up and down quark (or in the presence of electromagnetism) the global isospin symmetry is explicitly broken. As a consequence, the two charged $a^\pm$-mesons become pseudo-Goldstone bosons and acquire a small mass, similar to the pions. In contrast, the third component of isospin I_3 remains an exact symmetry even in the presence of quark masses and electromagnetism. Its spontaneous breaking by the dineutron condensate (30) necessarily leads to an exactly massless Goldstone boson and therefore to superfluidity. Second, also the baryon number B' is not conserved by the dineutron condensate. The only unbroken generator is $I_0 + \frac{1}{2}B'$ and corresponds to conserved electric charge Q. The simultaneous presence of three light pseudoscalars π and three light scalars a is a characteristic signal for our scenario with spontaneous breaking of the chiral and vector-like global $SU(2)$-symmetries at high density.

Protons and neutrons are presumably the lightest fermionic states in the vacuum of QCD_2. As the chemical potential for the conserved baryon number increases beyond a critical value the onset of the dineutron condensate triggers a true phase transition. The order parameter is related to the spontaneous breaking of the global symmetry with generator I_3. This is the "gas–liquid" transition to nuclear matter. If this phase transition would be of second order, it should belong to a universality class characterized by the breaking of $SU(2)$ to $U(1)$ (similar to $O(3)$-Heisenberg models) with small symmetry violations due to $m_u \neq m_d$. The fermionic fluctuations presumably play a role for this universality class. It is an open issue if for still higher values of the chemical potential another phase transition to high-density quark matter is connected to the changes in the isospin singlet diquarks shown in Table 5 or if the transition from nuclear to quark matter is continuous.

In the following we summarize the main features of the high-density phase transition for the case where spontaneous isospin symmetry breaking is absent or can be treated as a small correction.

(i) At high density complete color flavor locking is possible such that all gluons acquire a mass [9]. This can be realized by the same combination of two condensates as in the vacuum. If isospin is preserved there is no Goldstone boson and the high-density phase is not a superfluid. Chiral symmetry remains spontaneously broken at high density. Without spontaneous isospin breaking the transition between the vacuum and the high-density state of QCD_2 would not involve any change in the realization of symmetries if diquarks condense already in the vacuum. This leaves the possibilities of an analytic crossover or a first-order phase transition.

(ii) In an alternative scenario for the QCD vacuum the diquark condensate could vanish. In this case not all gluons can acquire a mass. There remains always an unbroken abelian $U(1)_c$ gauge symmetry which is part of $SU(3)_c$. Its gauge boson carries the quantum numbers of the ω-meson and remains massless. If, in the second scenario, the transition to QCD at high baryon density corresponds to condensation of a diquark which preserves isospin, one infers that the $U(1)_c$-gauge symmetry gets spontaneously broken in the transition to the high-density phase. The ω-meson acquires a mass. If isospin is conserved, the general characteristics of the high-density phase transition for the second scenario may resemble the transition for the abelian Higgs model [13]. This may lead to a first-order phase transition. The remarks on the limitations of the naive Higgs model from the preceeding section apply here as well. In particular, it is conceivable that no discontinuity occurs and the vacuum is analytically connected to the high-density state. The number of global abelian symmetries is the same with and without isospin singlet diquark condensates. Therefore no Goldstone boson is generated by an isospin-conserving diquark condensation in a transition to the high-density phase.

7 Gluodynamics

Finally, we turn to pure QCD without quarks. A description in the Higgs picture with spontaneous breaking of color may still be possible. Of course, it is not related to color-flavor locking any more, since there are no flavor symmetries. We present here a first attempt which demonstrates that suitable condensates can give a mass to all gluons. The resulting spectrum for the glueballs is, however, not very satisfactory. Our example should therefore not be interpreted as a proposal for the ground state of pure QCD but rather as an exploration in which directions one might go. For our first trial we follow the same philosophy as for the fermion bilinears and introduce scalar fields f_{ij} and s_{ijkl} for the color nonsinglets contained in $F^{\mu\nu}F_{\mu\nu}$ (cf. (23) and (26)). We assume that the effective action for these scalar fields,

$$\mathcal{L} = \frac{1}{2}Z_f(D^\mu f)_{ij}(D_\mu f)_{ji} + \frac{1}{2}Z_s(D^\mu s)_{ijkl}(D_\mu s)_{jilk} + U(f,s), \qquad (31)$$

has a potential U with minimum for non-zero expectation values of f and s (see [14] for a similar treatment of the color singlet). We want to demonstrate here that for suitable expectation values all gluons become massive and can be associated with vector glueballs.

We first consider the color octet $f_{ij} = (f^\dagger)_{ij}, f_{ii} = 0$. By appropriate $SU(3)$-transformations its vacuum expectation can be brought to the generic form

$$\langle f_{ij}\rangle = \bar{f}(\lambda_8)_{ij} + \bar{t}(\lambda_3)_{ij}. \qquad (32)$$

Such an expectation value cannot give mass to all gluons – the A_3- and A_8-vector mesons remain massless. For $\bar{t} \neq 0$ isospin symmetry is not conserved and the vector meson masses split according to

$$M_1^2 = M_2^2 = 2Z_f g^2 \bar{t}^2, \quad M_3^2 = 0, \qquad (33)$$
$$M_4^2 = M_5^2 = M_6^2 = M_7^2 = \frac{1}{2}Z_f g^2(3\bar{f}^2 + \bar{t}^2), \quad M_8^2 = 0.$$

Giving a mass to all gluons therefore needs one more non-vanishing expectation value and we consider here a particular direction in the 27-dimensional representation $s_{ijkl} = s^*_{jilk}$, namely

$$\langle s_{ijkl}\rangle = \bar{s}\left(\delta_{ik}\delta_{jl} - \frac{1}{3}\delta_{il}\delta_{jk}\right). \qquad (34)$$

This adds a mass term to the gluons corresponding to the symmetric Gell-Mann matrices

$$M_1^2 = M_3^2 = M_4^2 = M_6^2 = M_8^2 = 12Z_s g^2 \bar{s}^2,$$
$$M_2^2 = M_5^2 = M_7^2 = 0. \qquad (35)$$

Combining (35) with (34) all gluons have acquired a mass. We also observe the split between M_1^2 and M_2^2 etc.

We would like to associate the massive gluons with vector glueballs. Together with the scalar glueballs described by f, s and a corresponding color singlet they would be expected to dominate the low energy spectrum of pure QCD if the vacuum can be characterized by (32) and (34). (Spin 2 glueballs can be described by introducing additional fields for operators like $F_\mu{}^\rho F_{\rho\nu}$.) In order to differentiate between the discrete transformation properties of the glueballs we need the action of the parity transformation and charge conjugation

$$P: \quad A_0 \to A_0, \quad A_i \to -A_i, \quad C: \quad A_\mu \to -A_\mu^{\mathrm{T}}. \qquad (36)$$

where P is accompanied by a coordinate reflection. The "symmetric gluons" A_1, A_3, A_4, A_6, A_8 transform as 1^{--} vector glueballs, whereas the "antisymmetric gluons" A_2, A_5, A_7 correspond to 1^{-+} vector glueballs.

For the scalars we exploit

$$P: \; F^{ij} \to F^{ij}, \quad F^{0j} \to -F^{0j}, \quad f_{ij} \to f_{ij}, \quad s_{ijkl} \to s_{ijkl}, \qquad (37)$$
$$C: \; F^{\mu\nu} \to -(F^{\mathrm{T}})^{\mu\nu}, \quad f_{ij} \to f_{ji}, \quad s_{ijkl} \to s_{jilk}.$$

This shows that the expectation values (32) and (34) indeed conserve P and C. The scalar glueballs consist of 0^{++} and 0^{+-} states. The detailed mass spectrum of the vector and scalar glueballs depends on the properties of the effective potential $U(f,s)$ and may involve expectation values beyond those considered in (32) and (34). The effective action (31) with condensates (32) and (34) nevertheless predicts a selection rule, namely the absence of $1^{++}, 1^{+-}, 0^{-+}$ and 0^{--} states among the lightest glueball states. This has to be confronted with the results of lattice simulations [15] which find the lightest glueball states (with increasing mass) as $0^{++}, 0^{-+}, 2^{++}, 1^{+-}$. The lack of agreement shows that our first attempt has not been successful. Since the failure is linked to the action of the discrete symmetries P and C and the rotation group on the gauge fields A_μ, it concerns all other possible condensates which preserve these symmetries as well. An interesting way out of this dilemma may combine rotations with gauge symmetries or similar for the discrete symmetries, corresponding to a type of "color-Lorentz" or "color-spin" locking [16].[9]

Of course, it is not necessary that gluodynamics admits a Higgs description. It is well conceivable that the gluon propagator cannot be characterized by a mass term generated from "spontaneous symmetry breaking". This does not mean that the gluons will manifest themselves as massless excitations. The gluon propagator may simply admit no particle pole.[10] In any case, we expect that the mechanism which removes the perturbatively massless gluons from the very low momentum spectrum is very different for gluodynamics and for realistic QCD$_3$. We therefore emphasize that the role of the gluons and glueballs in realistic QCD$_3$ with three light quark flavors is quite distinct from gluodynamics. In our Higgs picture of realistic QCD the gluons are associated with the (ρ, K^*, ω)-mesons and do not correspond to glueball states. Glueballs play an important role in the low energy spectrum of gluodynamics, whereas for QCD$_3$ they may only appear as some higher excited states. In our picture the origin of this different role of the gluons and glueballs are the different patterns in the spontaneous breaking of the color symmetry.

8 Transition to gluodynamics and heavy quark potential

Our discussion of gluodynamics in the previous section raises the question what happens if the mass off all three light quarks is increased simultaneously. For simplicity we consider an equal mass m for the up, down and strange quarks. For large enough m the low momentum sector is characterized by gluodynamics. We therefore expect a transition from realistic QCD$_3$ to gluodynamics as m increases from small to large values. As we have discussed in the previous section this should be associated with a change of the effective infrared cut-off. Indeed, one may imagine that the infrared cut-off for QCD$_3$ provided by the octet induced gluon mass competes with some other, not yet well identified cut-off which is relevant for gluodynamics. We propose that for small enough m the octet condensate sets the largest IR-scale and the effective cut-off for gluodynamics is therefore ineffective. For example, a possible non-trivial momentum dependence of the gluon propagator for "massless" gluons may not get realized since the fluctuations responsible for it are cut off by the gluon mass term. On the other hand, for m above some critical value $m_{\rm cr}$ the IR cut-off of gluodynamics dominates. In turn, it may cut off the fluctuations that could be responsible for the octet condensate as, for example, the large size instantons [4]. For very large m the color octet $\bar{q}q$-bilinear simply plays no important role. The "switch" between the relevant cut-offs in some mass region around $m_{\rm cr}$ may be a sharp or smooth crossover since no global symmetries are affected. It would be interesting to have at least a rough idea about the value of $m_{\rm cr}$. It seems reasonable that pion and kaon masses below 200–300 MeV are already close to the chiral limit and therefore correspond to $m < m_{\rm cr}$. On the other hand, for both pseudoscalar and vector masses above 1 GeV one is presumably in the "heavy quark region" $m > m_{\rm cr}$.

Lattice simulations [12] show a rather smooth behavior of the spectrum if the pseudoscalar masses are varied between 300 MeV and 1 GeV. This is consistent with a smooth crossover and seems to exclude any phase transition between QCD$_3$ and gluodynamics. We find it not unlikely that the crossover region where the effects of octet condensation start to become important corresponds to pseudoscalar masses in the vicinity of 500 MeV. As a consequence, this mass region would not yet belong to the range where chiral perturbation theory applies. One may wonder where to expect the most prominent signs of the octet condensation once m decreases below $m_{\rm cr}$. One is perhaps the influence of the octet on the value of f_π. Another one concerns the quark mass dependence of the vector meson masses. The first involves the details of the interplay between the octet and singlet $\bar{q}q$ contribution as well as the quark mass dependence of the pseudoscalar wave function renormalization. The second needs an understanding of the mixing between gluons and $\bar{q}\gamma^\mu q$.

Another interesting issue is the shape of the heavy quark potential (for c or b quarks). Indeed, for m sufficiently large we expect a linearly rising potential $V(r)$ in a certain region of r, while for very large r the potential flattens due to string breaking. The slope of the linear rise is associated to the string tension. On the other hand, there may be a region of small m for which the linearly rising piece in the potential is absent. It is an interesting hypothesis that the potential could be described in this region by a naive Higgs picture with a Yukawa type potential

$$\frac{\partial V(r)}{\partial r} = \frac{4\hat{\alpha}_{\rm s}(r)}{3r^2}\exp(-\bar{M}_\rho r). \qquad (38)$$

Here $\bar{M}_\rho$ is the gluon mass associated to an average mass of the vector mesons (ρ, K^*, ω) and $\hat{\alpha}_{\rm s}(r)$ corresponds to

[9] See [17] for a discussion of color-spin locking for one-flavor QCD at high density

[10] These statements are meaningful only for some appropriate gauge fixing. On the other hand, it is not easy to see why a particle pole in a fixed gauge would not correspond to a physical particle in a gauge invariant setting

some suitably defined strong fine structure constant. The value of $\bar{M}_\rho$ depends on m and the precise shape of $\alpha_s(r)$ is influenced by $\bar{M}_\rho$ for the region $r \geq \bar{M}_\rho^{-1}$.

We point out that the string picture and the Yukawa potential are not incompatible – this reflects the duality between a confinement and a Higgs description. For a small enough quark mass m it is conceivable that $V(r)$ can both be described by the Yukawa potential (38) with reasonable $\hat{\alpha}_\mathrm{s}(r)$ and by a string-breaking picture. On the other hand, for large m the Yukawa description is expected to break down and only the string picture remains valid. In the limit $m \to \infty$ one then recovers gluodynamics without string breaking.

Let us first discuss the qualitative behavior of $V(r)$ for very large r in the string picture. For finite m there will be a "string-breaking scale" r_B^{-1} such that the potential approaches quickly a constant for $r > r_\mathrm{B}$. The approach to the constant is expected to be exponential:

$$\frac{\partial V}{\partial r} = \sum_i c_i(r) \exp(-M_i r). \tag{39}$$

For large enough r (39) will be dominated by the masses M_i of the lightest mesons which are exchanged between heavy quarks (or heavy charmed or beauty baryons or mesons). Neglecting the pions and other pseudoscalar mesons (whose contribution should also be added to (38)) this is precisely the behavior of (38) with $M_i = \bar{M}_\rho$. For very large r beyond the string-breaking distance r_B we expect always a Yukawa type force, independent of the precise mechanism generating the meson masses. This region in the potential therefore finds a similar description in the Higgs picture and the string-breaking picture.

A Yukawa type description is excluded, however, if the string-breaking distance r_B is large enough, say $r_\mathrm{B}^{-1} \gtrsim$ 300 MeV or $r_\mathrm{B} \gtrsim 0.7\,\mathrm{fm}$. Here we define r_B as the upper end of a range $r < r_\mathrm{B}$ for which $\partial V/\partial r \approx \sigma$, with σ the constant string tension. In fact, for $\bar{M}_\rho r_\mathrm{B} \gg 1$ one concludes that there exists a range in r where the potential is linearly rising and simultaneously $\bar{M}_\rho r \gg 1$. This is in contradiction to the exponential suppression for the "pure Yukawa potential" (38). On the other hand, for $r_\mathrm{B}^{-1} \to$ 400 MeV the linear piece in the potential disappears. In this range of r the exponential suppression due to the mass is not yet dominant and the naive Higgs picture with Yukawa potential (38) could become valid.

For large enough m the string-breaking distance r_B can be roughly estimated from a simple energy argument. The potential energy for the separation of two heavy quarks with mass m_H is $V = \sigma r$. The string will break once $2m_H + \sigma r$ is large enough in order to produce twice the mass m_B of a heavy-light meson (e.g. D- or B-meson). This yields

$$r_\mathrm{B} \approx \frac{2(m_B - m_H)}{\sigma}. \tag{40}$$

For very large m one has $m_B \approx m_H + m$ and therefore $\bar{M}_\rho r_\mathrm{B} = 2m\bar{M}_\rho/\sigma$. Simplifying further $\bar{M}_\rho \approx 2m$ yields as a condition for $m > m_\mathrm{cr}$ the inequality $2m \gg \sqrt{\sigma}$ or $m \gg 200$ MeV. This simple estimate may become valid for $m \gtrsim 500\,\mathrm{MeV}$. Present lattice simulations with pion masses above 300–400 MeV have seen no sign of string breaking yet ($r_\mathrm{B} \gtrsim 1.5\,\mathrm{fm}$). On the other hand, for small m the difference $m_B - m_H \approx a\Lambda_\mathrm{QCD}$ is dominated by gluonic effects that remain non-zero even in the chiral limit $m \to 0$. For $m_\mathrm{B} - m_H \approx$ (200–500) MeV, which may be a realistic range, one finds $r_\mathrm{B}^{-1} \approx$ (500–200) MeV. For the upper value $r_\mathrm{B}^{-1} \approx 500\,\mathrm{MeV}$ it seems questionable if one can observe a linearly rising piece of the heavy quark potential at all, whereas for $r_\mathrm{B}^{-1} \approx$ 200 MeV this should be possible. We conclude that it is indeed conceivable that for realistic QCD$_3$ a Yukawa description (38) becomes valid. The possible dual description of the heavy quark potential either by string breaking or by a Yukawa potential adds another facet to the postulated duality between the Higgs and confinement pictures.

It may be an interesting task for future lattice simulations to find out how the string-breaking distance r_B depends on the quark mass m. It seems also worthwhile to study for which shape of $\hat{\alpha}_\mathrm{s}(r)$ in (38) one can achieve agreement with phenomenological constraints on the heavy quark potential. In momentum space the quantity

$$\tilde{\alpha}(q^2) = \alpha_v(q^2)\frac{q^2}{q^2 + \bar{M}_\rho^2(q^2)}, \tag{41}$$

should be close to

$$\tilde{\alpha}_\mathrm{R}(q^2) = \frac{4\pi}{9}\frac{1}{\ln(1 + q^2/\Lambda_\mathrm{R}^2)}, \tag{42}$$

in the momentum range relevant for charmonia. Here $\tilde{\alpha}_\mathrm{R}$ corresponds to the phenomenologically successful Richardson potential [18] with $\Lambda_\mathrm{R} = 400\,\mathrm{MeV}$. One may also have to take into account that due to the running of the gauge coupling the effective gluon mass $\bar{M}_\rho(q^2)$ will be momentum dependent.

9 Conclusions

We have presented a possible Higgs description of the vacuum of two-flavor QCD$_2$ in terms of $\bar{q}q$ and qq-condensates. Isospin and baryon number are conserved and electric charges are integer. The quarks can be associated with baryons and the gluons with vector mesons. Most strikingly, one finds excitations with the quantum numbers of strange mesons (K^*) and strange baryons (Σ, Λ) in the spectrum of QCD$_2$.

In this picture the gluons acquire a mass by the Higgs mechanism. We find that an isospin singlet vector meson (ω) remains massless in the absence of a diquark condensate. This contrasts with three-flavor QCD$_3$. However, QCD$_2$ admits also a diquark condensate in the vacuum which can give a mass to the ω-meson [8]. Despite the spontaneous breaking of the "standard" baryon number B by the diquark condensate, there remains a new conserved baryon number B'. The "hyperon-like states" Σ and Λ are neutral with respect to B'. In addition, nontrivial gluon condensates are possible in QCD$_2$. All these features differ from QCD_3, and we speculate about a phase transition from QCD_3 to QCD_2 as the strange quark mass increases beyond a critical value.

Two-flavor QCD_2 cannot be tested directly by experimental observation. On the other hand, QCD_2 can be simulated in lattice calculations, and this may provide important tests for our picture. The possible tests concern both the properties of the vacuum and, perhaps at a later stage, the transition to the high-density state. The most striking signal would be a phase transition in the vacuum properties as a function of the strange quark mass. The characteristics of the high-density transition in QCD_2 depend crucially on the vacuum properties.

We also have made a first attempt to understand the vacuum properties of gluodynamics (without light quarks) in terms of colored gluonic condensates. The partial failure of scalar condensates to reproduce an acceptable glueball spectrum still leaves open the possibility that a type of color-spin locking [16] may offer an interesting alternative. In this scenario a residual rotation symmetry is composed of ordinary rotations accompanied by gauge transformations. It seems not completely excluded that such a color-spin locking condensate could also occur in QCD_2 and perhaps give a mass to the ω-meson even in the absence of a diquark condensate. As an (perhaps more likely) alternative gluo-dynamics admits no Higgs picture and spontaneous color breaking occurs only in the presence of sufficiently light quarks. For all the various alternatives investigated in this note we have found interesting qualitative changes in the transition from QCD_3 to QCD_2. We suggest that lattice simulations could clarify this interesting issue.

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References

1. C. Wetterich, Phys. Lett. B **462**, 64 (1999), hep-th/9906062; Phys. Rev. D **64**, 036003 (2001), hep-ph/0008150
2. T. Banks, E. Rabinovici, Nucl. Phys. B **160**, 349 (1979); E. Fradkin, S. Shenker, Phys. Rev. D **19**, 3682 (1979); G. 't Hooft, in Recent Developments in Gauge Theories (Plenum, New York 1980) p. 135; S. Dimopoulos, S. Raby, L. Susskind, Nucl. Phys. B **173**, 208 (1980); T. Matsumoto, Phys. Lett. B **97**, 131 (1980); M. Yasuè, Phys. Rev. D **42**, 3169 (1990)
3. M. Alford, K. Rajagopal, F. Wilczek, Nucl. Phys. B **537**, 443 (1999); T. Schäfer, F. Wilczek, Phys. Rev. Lett. **82**, 3956 (1999); M. Alford, J. Berges, K. Rajagopal, Nucl. Phys. B **558**, 219 (1999)
4. C. Wetterich, hep-ph/0012013, to appear in Phys. Lett. B
5. C. Wetterich, Eur. Phys. J. C **18**, 577 (2001), hep-ph/9908514
6. C. Wetterich, hep-ph/0102044
7. T. Schäfer, Phys. Rev. D **64**, 037501 (2001), hep-lat/0102007
8. J. Berges, C. Wetterich, Phys. Lett. B **512**, 85 (2001), hep-ph/0012311
9. J. Berges, Phys. Rev. D **64**, 014010 (2001), hep-ph/0012013
10. S.C. Frautschi, in Proceedings of the Workshop on Hadronic Matter at Extreme Energy Density, edited by N. Cabibbo, Erice, 1978; B.C. Barrois, Nucl. Phys. B **129**, 390 (1977); D. Bailin, A. Love, Phys. Rep. **107**, 325 (1984); M. Alford, K. Rajagopal, F. Wilczek, Phys. Lett. B **422**, 247 (1998); R. Rapp, T. Schäfer, E. Shuryak, M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998); J. Berges, K. Rajagopal, Nucl. Phys. B **538**, 214 (1999); M. Alford, J. Berges, K. Rajagopal, Nucl. Phys. B **558**, 219 (1999); N. Evens, S. Hsu, M. Schwetz, Nucl. Phys. B **551**, 275 (1999); R. Pisarski, nucl-th/9912070; D.T. Son, Phys. Rev. D **59**, 094019 (1999); R. Pisarski, D. Rischke, Phys. Rev. D **60**, 114033 (1999); W. Brown, J. Liu, H. Ren, Phys. Rev. D **62**, 054016 (2000); T. Schäfer, E. Shuryak, nucl-th/0010049
11. K. Bardakci, M.B. Halpern, Phys. Rev. D **6**, 696 (1972); R. Mohapatra, J. Pati, A. Salam, Phys. Rev. D **13**, 1733 (1976); A. De Rujula, R. Giles, R. Jaffe, Phys. Rev. D **17**, 285 (1978); B. Iijima, R. Jaffe, Phys. Rev. D **24**, 177 (1981)
12. C. Bernard et al., hep-lat/0104002; A. Ali Khan et al., hep-lat/0105015; T. Kaneko, hep-lat/0111005; Y. Namekawa et al., hep-lat/0209073
13. B. Bergerhoff, F. Freire, D. Litim, S. Lola, C. Wetterich, Phys. Rev. B **53**, 5734 (1996), hep-ph/9503334; K. Kajantie, M. Karjalainen, M. Laine, J. Peisa, Nucl. Phys. B **520**, 345 (1998)
14. M. Reuter, C. Wetterich, Phys. Rev. D **56**, 7893 (1997)
15. See, e.g., M. Teper, hep-th/9812187
16. M. Reuter, C. Wetterich, Phys. Lett. B **334**, 412 (1994), hep-ph/9405300
17. T. Schäfer, Phys. Rev. D **62**, 094007 (2000)
18. J.L. Richardson, Phys. Lett. B **82**, 272 (1979)